Things to Remember

When n is a positive integer :

01. $(a+x)^n = a^n + {}^nC_1a^{n-1}x + {}^nC_2a^{n-2}x^2 + \cdots + {}^nC_ra^{n-r}x^r + \cdots + x^n$ $\left[t_r = {}^nC_ra^{n-r}x^r\right]$

02. $(1+x)^n = 1 + {}^nC_1x + {}^nC_2x^2 + \cdots + {}^nC_rx^r + \cdots + x^n$ $\left[t_r = {}^nC_rx^r\right]$

03. $(a-x)^n = a^n - {}^nC_1a^{n-1}x + {}^nC_2a^{n-2}x^2 - \cdots + (-1)^n x^n$

04. $(1-x)^n = 1 - {}^nC_1x + {}^nC_2x^2 - \cdots + (-1)^n x^n$

05. Middle Term of $(1+x)^n$:

(a) when n is even M.T. $= {}^nC_{n/2}x^{n/2}$ (b) when n is odd M.T. $= {}^nC_{\frac{n-1}{2}}\, x^{\frac{n-1}{2}}$ and ${}^nC_{\frac{n+1}{2}}\, x^{\frac{n+1}{2}}$

06. Greatest Co-efficient of $(a+x)^n$ or $(1+x)^n$:

(a) when n is even G.C. $= \left(\frac{n}{2}+1\right)$th i.e., middle term's co-efficient.

(b) when n is odd G.C. $= \left(\frac{n-1}{2}+1\right)$th or $\left(\frac{n+1}{2}+1\right)$th term has the greatest co-efficient, both being equal.

07. Greatest Term of $(a+x)^n$ when x, a > 0.

$$\frac{t_{r+1}}{t_r} = \frac{n-r+1}{r}\cdot\frac{x}{a}$$

$t_{r+1} >=< t_r$ according as $r <=> \frac{n+1}{x+a}x$.

08. Properties : (a) $C_0 + C_1 + C_2 \cdots + C_n = 2^n$. (b) $C_1 + C_2 + C_3 \cdots + C_n = 2^n - 1$.

(c) $C_0 + C_2 + C_4 \cdots = C_1 + C_3 + C_5 \cdots = 2^{n-1}$.

Infinite Binomial

09. Expansion for any value of n (n ≠ a positive integer)

(a) if $a^2 > x^2$ then $(a+x)^n = a^n + {}^nC_1a^{n-1}x + {}^nC_2a^{n-2}x^2 + \cdots \infty$

(b) if $a^2 < x^2$ then $(a+x)^n = x^n + {}^nC_1x^{n-1}a + {}^nC_2x^{n-2}a^2 + \cdots \infty$

10. Deduction : n ≠ a positive integer $[-1 < x < 1$ or $|x| < 1]$

(a) $(1+x)^n = 1 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + \cdots \infty$ (b) $(1-x)^n = 1 - {}^nC_1x + {}^nC_2x^2 - {}^nC_3x^3 + \cdots \infty$

For : $-n \neq$ a positive integer

(c) $(1-x)^{-n} = 1 - {}^nC_1x + {}^nC_2x^2 - {}^nC_3x^3 + \cdots \infty$ (d) $(1-x)^{-n} = 1 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + \cdots \infty$

11. Important Results : when $-1 < x < 1$

(a) $(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots \infty$

(b) $(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots \infty$

(c) $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \cdots \infty$

(d) $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots \infty$

(e) $(1+x)^{-3} = 1 - 3x + 6x^2 - 10x^3 + \cdots \infty$

(f) $(1-x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + \cdots \infty$

(g) $(1-x^2)^{-\frac{1}{2}} = 1 + \frac{1}{2}x^2 + \frac{1.3}{2.4}x^4 + \frac{1.3.5}{2.4.6}x^6 + \cdots \infty$

Sum - HS

01. Obtain the term free from x in the expansion $\left(2x + \frac{1}{3x^2}\right)^9$ **H-88,78,Taki,SCH-88,Joy-89,Rah-86**

02. Find the co-efficient of x in $(1 - 2x^3 + 3x^5)\left(1 + \frac{1}{x}\right)^{10}$ **H-79,Ashu-83,Jaga-84,PBh-85,Taki,Xav-86,PBh-88**

03. If $-1 < x < 1$, prove that **H-02,85,79,Xav,SPt-87,Jaga-84,Ashu-83**

$$(1 + x + x^2 + x^3 + \cdots \infty)(1 + 2x + 3x^2 + 4x^3 + \cdots \infty) = \frac{1}{2}(1.2 + 2.3x + 3.4x^2 + \cdots \infty)$$

04. Find the co-efficient of x^r in the expansion $\frac{1+x}{(1-x)^2}$ **H-80,SCh-83**

05. Find the co-efficient of x^{16} in the expansion $x^{10}(x-2)^{10}$. **H-81,SCh-86,Taki-87**

06. Given that $(1-x)^n = C_0 - C_1x + C_2x^2 - C_3x^3 + \cdots\cdots + (-1)^n\, C_r x^r + \cdots\cdots + (-1)^n\, x^n$; show that $C_1 + 2C_2 + 3C_3 \cdots\cdots + nC_n = n.2^{n-1}$. **J-82, H-83,81,Viv,Rah-88,Jaga,SCh-86,Beth-85,Hare-83**

07. Find the first three terms in the expansion of $(2+3x)^{-3}$ and state the condition of validity of the expansion. **H-81,Belur-88,87,Rah-86**

08. Write down the first four terms in the infinite series expansion of $(1+x)^{1/4}$ in ascending powers of x and state the condition of validity of the expansion. **H-82,Xav-87,Viv-88**

09. Find the term independent of x in the expansion of $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$ **J-84, H-90,82,City-87,SPt-88,Nar,SCh-86,Ashu-83**

10. Prove that the co-efficient of the middle term in the expansion of $(1+x)^{12}$ is equal to the sum of the co-efficients of the two middle terms in the expansion of $(1+x)^{11}$. **H-84,82**

11. Find the middle term in the expansion of $\left(\frac{x^2}{3} + \frac{3}{x^3}\right)^8$ **H-83,Rah-87,Xav-86,85,83,Ashu-85,Joy-83**

12. If in the expansion of $(1+x)^{32}$, the co-efficient of the $(3r+1)$th termbe equal to the co-efficient of the $(r+5)$th term, show that the value of r is 7. **J-75,H-83, Uttar,Jaga-86,AV-83**

13. If $|x| < \frac{1}{2}$, find the co-efficient of x^n in the expansion of $(1-2x)^{-\frac{1}{2}}$ **H-83**

14. Which term in the expansion of $(1+x)^p\left(1+\frac{1}{x}\right)^q$ is independent of x, where p and q are positive integers ? What is the value of the term ? **H-89,84,Bally,Mitra-88,Hindu-86, Hare-85**

15. If n is a positive integer and $|x| < 1$, show that the co-efficient of the nth term in the expansion of $(1-x)^{-n}$ is twice the co-efficient of the $(n-1)$th term. **H-84,Xav-85**

16. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \cdots\cdots + C_nx^n$, prove that $C_0 + 2C_1 + 3C_2 + \cdots\cdots + (n+1)C_n = (n+2)2^{n-1}$. **H-85,Joy-89,City-88,Rah-86,SCh-85**

17. Find the co-efficient of x^{-2} in the expansion of $\left(3x - \frac{7}{x}\right)^8$. **H-86,Viv-86,Nar,Bally-83**

18. Write down the first four terms in the expansion of $(2+x)^{-2}$ in ascending power of $\frac{1}{x}$, stating the condition of validity of the expansion. **H-86**

19. Use binomial theorem to show that if $n \geq 2$ is an integer, then $11^n - 10n - 1$ is **divisible** by 100. **H-86, PBh-88**

19a. Use binomial theorem to show that if $n \geq 2$ is an integer, then $3^{2n+2} - 8n - 9$ is **divisible** by 64. **IIT-77, HS-96**

20. Use binomial theorem to calculate $(1{\cdot}02)^8$ correct to two places of decimals. **HS-87, Ashu-85**

21. Examine whether there is any term containing x^9 in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{20}$. **HS-87**

22. If $y = 2x + 3x^2 + 4x^3 + \cdots\cdots \infty$ for $|x| < 1, |y| < 1$, find the first three terms in the expansion of x in a series of ascending powers of y. **HS-87, AV-83**

23. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \cdots\cdots + C_nx^n$, show that

$$C_0C_n + C_1C_{n-1} + C_2C_{n-2} + \cdots\cdots + C_nC_0 = \frac{\lfloor 2n}{(\lfloor n)^2}$$ **H-88**

24. The condition of validity of the expansion being assumed to be satisfied, find the fifth term in the expansion of $(1-2x)^{-\frac{1}{2}}$ in ascending powers of x. [give your answer in the simplified form]. **J-84,H-88,Nar-85**

25. Express the series $2 + \frac{5}{\lfloor 2 . 3} + \frac{5.7}{\lfloor 3 . 3^2} + \frac{5.7.9}{\lfloor 4 . 3^3} + \cdots\cdots$ to ∞ in the form of a binomial expansion and hence find its value. **J-80, SCh-85**

26. Find the **numerically greatest term** (or terms) in the expansion of $(2-3x)^{13}$ when $x = ½$ **PBh,Belur,Uttar-88 PBh-87, SCh-86**

27. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \cdots\cdots + C_nx^n$, prove that

$C_0 + 3C_1 + 5C_2 + \cdots\cdots + (2n+1)C_n = (n+1)2^n$. **Taki-87,85, Viv-83**

28. If $|a| < 1$ and $|b| < 1$, then find the sum of the series **:**

$$1 + (1+a)b + (1+a+a^2)b^2 + (1+a+a^2+a^3)b^3 + \cdots\cdots\ \cdots\cdots$$

29. The successive co-efficients in the expansion of $(1+x)^n$ is a, b, c, show that $n = \frac{2ac + b(a+c)}{b^2 - ac}$.

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Sum - HP

01. If in the expansion of $(1+ax+2x^2)^6$ the co-efficients of x^2 and x^{11} be 27 and -192 respectively, then prove that $a=-1$. **J-66**

02. Sum to infinity : $1+\frac{1}{2}\cdot\left(\frac{2}{3}\right)+\frac{1}{2}\cdot\frac{3}{4}\left(\frac{2}{3}\right)^2+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\left(\frac{2}{3}\right)^3+\cdots\cdots\infty$ **J-66,Joy-86**

03. Sum to infinity : $\frac{1}{2}+\frac{1\cdot3}{2\cdot4}\cdot\left(\frac{1}{4}\right)+\frac{1\cdot3\cdot5}{2\cdot4\cdot6}\cdot\left(\frac{1}{4}\right)^2+\cdots\cdots\infty$ **J-68,Taki-88,Nar-87**

04. If $(1+x)^n=C_0+C_1x+C_2x^2+\cdots\cdots+C_nx^n$, show that $\frac{C_0}{1}+\frac{C_1}{2}+\frac{C_2}{3}+\cdots\cdots+\frac{C_n}{n+1}=\frac{2^{n+1}-1}{n+1}$.

H-98,95, J-82,69, I-75, SPt-06,Bally, Kanai-88,Hind,PBh-87Joy-86,Hind,City-85,Nar,SCh-83

04(a). If $(1+x)^n=C_0+C_1x+C_2x^2+\cdots\cdots+C_nx^n$, show that

$$\frac{C_0}{1}-\frac{C_1}{2}+\frac{C_2}{3}-\cdots\cdots+(-1)^n\frac{C_n}{n+1}=\frac{1}{n+1}.$$ **SPt-01**

05. If P_n be the product of all the co-efficients in the expansion of $(1+x)^n$, then prove that $\frac{P_{n+1}}{P_n}=\frac{(n+1)^n}{\underline{|n}}$.

J-70, Mitra-88,Nar-86,PBh-85,Hooghly-84,Taki-83

06. In the expansion of $(1+x)^n$, if the numerical co-efficients of any four consecutive terms be a_1, a_2, a_3, a_4, then prove that $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}+\frac{2a_2}{a_2+a_3}$. **J-71, I-75**

07. If c be such a number that c^4 can be neglected in comparison with l^4, then show that

$$\sqrt{\frac{l}{l+c}}+\sqrt{\frac{l}{l-c}}=2+\frac{3}{4}\cdot\frac{c^2}{l^2}$$ **J-74, PBh-87,83**

08. Find the index of the power of x in the greatest co-efficient in the expansion of $\left(2+\frac{5x}{2}\right)^{12}$ **J-76**

09. If $y=3x+6x^2+10x^3+\cdots\cdots\infty$ then prove that $x=\frac{y}{3}-\frac{1\cdot4}{3^2\cdot\underline{|2}}y^2+\frac{1\cdot4\cdot7}{3^3\cdot\underline{|3}}y^3-\cdots\cdots$ to ∞ **J-85**

10. If $(1+x)^n=C_0+C_1x+C_2x^2+\cdots\cdots+C_nx^n$, show that

$$(C_0+C_1)(C_1+C_2)(C_2+C_3)\cdots\cdots(C_{n-1}+C_n)=\frac{(n+1)^n}{\underline{|n}}C_1C_2C_3\cdots\cdots C_n$$ **J-86, Nar-88,87,Hind-84**

11. Sum to infinity : $\frac{1}{4}+\frac{1\cdot3}{4\cdot6}+\frac{1\cdot3\cdot5}{4\cdot6\cdot8}+\cdots\cdots\infty$ [1] **Nar-86**

12. If $x=1+a+a^2+\cdots\cdots\cdots$ to ∞ and $y=1+b+b^2+\cdots\cdots\cdots$ to ∞, where a and b are proper fractions, then the value of $z=1+ab+a^2b^2+\cdots\cdots\cdots$ to ∞ is given by (i) $\frac{xy}{x+y+1}$ (ii) $\frac{xy}{x+y-1}$ (iii) $\frac{xy}{x-y+1}$ (iv) $\frac{xy}{x-y-1}$. **Bri, Hindu-87,83**

13. If n be a positive integer, and if the third, fourth, fifth and sixth terms in the expansion of $(x+p)^n$ be a, b, c, d respectively, prove that $\frac{b^2-ac}{c^2-bd}=\frac{5a}{3c}$. **Hindu-88,86**

14. If $(1+x)^n=C_0+C_1x+C_2x^2+\cdots\cdots+C_nx^n$, show that $C_1^2+2C_2^2+3C_3^2+\cdots\cdots+nC_n^2=\frac{\underline{|2n-1}}{\left(\underline{|n-1}\right)^2}$ **J-90**

15. Sum to infinity : $1+\frac{1}{4}+\frac{1\cdot3}{4\cdot8}+\frac{1\cdot3\cdot5}{4\cdot8\cdot12}+\cdots\cdots\infty$ [$\sqrt{2}$] **J-07,H-89,69,67,Mitra-88,City-86,SCh-85,83**

16. If $(1+x)^n=C_0+C_1x+C_2x^2+\cdots\cdots+C_nx^n$, prove that

$C_1-2C_2+3C_3-\cdots\cdots+(-1)^{n-1}n\cdot C_n=0$. **H-72,Moulana-88, SCh-86**

17. If $y=x+x^2+2x^3+\cdots\cdots\cdots+\frac{\underline{|2n}}{\underline{|n}\,\underline{|n+1}}x^{n+1}+\cdots\cdots\cdots$, prove that $y^2-y+x=0$

18. Show that the middle term of the expansion $(1-x)^{2n}$ is $\frac{(-1)^n\cdot1\cdot3\cdot5\cdot7\cdots\cdots(2n-1)}{\underline{|n}}\cdot2^n\cdot x^n$ **H-89,Vidyasagar-85**

19. Sum to infinity : $1+\frac{1}{6}+\frac{1\cdot4}{6\cdot12}+\frac{1\cdot4\cdot7}{6\cdot12\cdot18}+\cdots\cdots\infty$ [$\sqrt[3]{2}$] **SPt-87,84, Xav,Jaga-86**

20. Sum to infinity : $1+\frac{10}{9}+\frac{10.16}{9.18}+\frac{10.16.22}{9.18.27}+\cdots\cdots\infty$ [$3\sqrt[3]{9}$] **Hind-87, Rah-86**

21. Sum to infinity : $1+\frac{4}{6}+\frac{4.5}{6.9}+\frac{4.5.6}{6.9.12}+\cdots\cdots\infty$ $\left[\frac{19}{8}\right]$ **Narendranath-83**

22. Show that $\sqrt{2}=\frac{7}{5}\left\{1+\frac{1}{10^2}+\frac{1.3}{1.2}\cdot\frac{1}{10^4}+\frac{1.3.5}{1.2.3}\cdot\frac{1}{10^6}+\cdots\cdots\infty\right\}$ **Hind-83**

23. If $y=x-x^2+x^3-x^4+\cdots\cdots\cdots\infty$ when $-1<x<1$, prove that $x=\frac{y}{1-y}$ **Bally-83**

24. Find the co-efficient of x^n in the expansion of $\frac{x}{(1-2x)(1-3x)}$. **SPt-86,StL-88**

25. If 3 consecutive co-efficients in the expnsion of $(1+x)^n$ be 66, 220, 495, find the value of n. [12] **PBh-87,StL-88**

26. Show that $\sqrt{\frac{1+x}{1-x}}=1+\left(\frac{x}{1+x}\right)+\frac{3}{2}\cdot\left(\frac{x}{1+x}\right)^2+\frac{5}{2}\cdot\left(\frac{x}{1+x}\right)^3+\cdots\cdots\infty$ **Hind-88,86**

27. If x is so small that x^2 and higher power of x may be neglected, show that $\frac{\sqrt{1+x}+\sqrt[3]{(1+x)^2}}{1+x+\sqrt{1+x}}=1-\frac{x}{6}$. **J-73**

28. Show that $(x-1)^2$ is a factor of $x^n-nx+n-1$.

29. Find the term independent of x in the expansion of $(1+x)^m\left(1-\frac{1}{x}\right)^n$, where m and n are positive integers ?

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Sum - TP

01. If $C_0, C_1, C_2, C_3, \cdots\cdots C_{15}$ are the binomial co-efficients in the expansion of $(1+x)^{15}$, prove that
$\frac{C_1}{C_0}+2\cdot\frac{C_2}{C_1}+3\cdot\frac{C_2}{C_1}+\cdots\cdots+15\cdot\frac{C_{15}}{C_{14}}=120.$ **SPt-02, I-61,Xav-85,Uttar-88**

02. Evaluate $(0\cdot99)^{15}$ correct to four places of decimals. **I-64,Xav-88 [T-02]**

03. Find the **term independent** of x in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{\sqrt{3}}{2x^2}\right)^{10}$ **I-65, Xav-88 [T-04]**

04. If the co-efficients of $(2r+4)$th, $(r-2)$th terms in the expansion of $(1+x)^{18}$ are equal, find r. **I-64, Law-88 [T-05]**

05. Find the value of **greatest term** in the expansion of $\sqrt{3}\left(1+\frac{1}{\sqrt{3}}\right)^{20}$ **I-65, PBh-88 [T-06]**

06. If $(1+x)^{15}=C_0+C_1x+C_2x^2+\cdots\cdots+C_{15}x^{15}$, find the value of
$C_2+2C_3+3C_4\cdots\cdots+14C_{15}$ [212993] **I-66 [T-07]**

07. If the co-efficients of 2nd, 3rd and 4th terms in the expansion of $(1+x)^{2n}$ are in A.P.
show that $2n^2-9n+7=0$. **J-07,I-68,Law-87 [T-09]**

08. Find the **term independent** of x in the expansion of $(1+x+x^2)\left(\frac{3}{2}x^2+\frac{1}{3x}\right)^9$ **I-62, Hind-87 [T-12]**

09. If $C_0, C_1, C_2, C_3, \cdots\cdots C_n$ be the co-efficients in the expansion of $(1+x)^n$, prove that

(a) $C_0^2+C_1^2+C_2^2+\cdots\cdots+C_n^2=\frac{\lfloor 2n}{(\lfloor n)^2}$ **H-00,97, I-73, Vidya-85 [T-14]**

(b) $C_0C_r+C_1C_{r+1}+C_2C_{r+2}+\cdots\cdots+C_nC_{n-r}=\frac{\lfloor 2n}{\lfloor n+r\cdot\lfloor n-r}$ **I-74, Belur-87,86, Hind-83 [T-14]**

10. Show that the larger of $(99)^{50}+(100)^{50}$ and $(101)^{50}$ is $(101)^{50}$ **I-82 [T-19]**

11. Show that the sum of the co-efficients of the polynomial $(1+x-3x^2)^{2143}$ is (-1). **I-82 [T-19]**

12. Find the **term independent** of x in $(1+x)^2\left(x-\frac{1}{x}\right)^7$ [70] **[K-41]**

13. Show that the **middle term** in the expansion of $\left(x-\frac{1}{x}\right)^{2n}$ is $\frac{1.3.5\ldots\ldots\ldots(2n-1)}{n!}(-2)^n$. **[K-42]**